Analisi Matematica II

Politecnico di Milano – Ingegneria

Esercizi – Equazioni differenziali lineari del secondo ordine

1. Determinare l'integrale generale delle seguenti equazioni differenziali lineari omogenee.
   (a) $y'' - 7y' + 6y = 0$
   (b) $y'' + 6y' - 9y = 0$
   (c) $y'' - 6y' + 9y = 0$
   (d) $4y'' + 4\sqrt{3}y' + 3y = 0$
   (e) $y'' - 4y' + 8y = 0$
   (f) $2y'' - 6y' + 5y = 0$
   (g) $y'' + y' + y = 0$

2. Determinare la soluzione dei seguenti problemi di Cauchy.
   (a) $y'' + 4y' + 4y = 0,\ \ y(0) = 2\,,\ \ y'(0) = 0$
   (b) $y'' - 2y' + 9y = 0,\ \ y(0) = 1\,,\ \ y'(0) = -1$
   (c) $y'' + 5y' + 4y = 0,\ \ y(1) = 0\,,\ \ y'(1) = 1$
   (d) $y'' + 5y' + 7y = 0,\ \ y(0) = -1\,,\ \ y'(0) = -1$
   (e) $y'' - 6y' + 34y = 0,\ \ y(0) = 1\,,\ \ y'(0) = -1$

3. Riducendo l'ordine dell'equazione, determinare l'integrale generale delle seguenti equazioni differenziali lineari complete.
   (a) $y'' - 2y' = 2x + 3x^2$
   (b) $y'' - 3y' = x\mathrm{e}^x$
   (c) $y'' + y' = \dfrac{1}{1 + \mathrm{e}^x}$
   (d) $y'' - y' = \sin x + x \cos x$

4. Usando il metodo di somiglianza, determinare l'integrale generale delle seguenti equazioni differenziali lineari complete.
   (a) $y'' - 7y' + 6y = 2 + 16x - 16x^2$
   (b) $y'' + 5y' = 7 + 10x$
   (c) $y'' + 9y = 2 + 9x^2$
   (d) $y'' + 7y' + 12y = 20\,\mathrm{e}^x$
   (e) $y'' + 7y' + 12y = 20\,\mathrm{e}^{-3x}$
   (f) $y'' + 2y = 6\,\mathrm{e}^x$
   (g) $y'' - y' - 6y = 17\,\mathrm{e}^{2x} \cos x$
   (h) $y'' - 2y' + 5y = 4\,\mathrm{e}^x \sin 2x$
   (i) $y'' + 3y = \sin \sqrt{3}x$
   (j) $y'' - y' + y = \mathrm{e}^x$

5. Usando il metodo di variazione delle costanti arbitrarie, determinare l'integrale generale delle seguenti equazioni differenziali lineari complete.
   (a) $y'' - 6y' + 9y = \dfrac{\mathrm{e}^{3x}}{1 + x^2}$
   (b) $y'' - y = \mathrm{e}^{2x} \cos \mathrm{e}^x$

(c) $y'' - 4y' + 3y = \dfrac{e^{2x}}{1 + e^x}$

(d) $y'' - 5y' + 6y = \dfrac{1}{1 + e^{2x}}$

(e) $y'' - 9y' + 18y = e^{e^{-3x}}$

6. Determinare la soluzione dei seguenti problemi di Cauchy.

(a) $y'' - 2y' + y = x\, e^{2x}\,,\ \ y(0) = 2\,,\ \ y'(0) = 1$

(b) $y'' + 6y' + 8y = 4x\, e^{-2x}\,,\ \ y(0) = 1\,,\ \ y'(0) = 1$

(c) $y'' + 10y' + 25y = x\, e^{-x}\,,\ \ y(0) = 0\,,\ \ y'(0) = 0$

(d) $y'' + 4y = 8x^2\,,\ \ y(0) = 0\,,\ \ y'(0) = 4$

(e) $y'' + 3y = \sin x\,,\ \ y(0) = 0\,,\ \ y'(0) = 1$

Risposte

1. (a) $y(x) = c_1\,e^{x} + c_2\,e^{6x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (b) $y(x) = c_1\,e^{(-3+3\sqrt{2})x} + c_2\,e^{(-3-3\sqrt{2})x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (c) $y(x) = (c_1 + c_2 x)\,e^{3x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (d) $y(x) = (c_1 + c_2 x)\,e^{-\frac{\sqrt{3}}{2}x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (e) $y(x) = c_1\,e^{2x}\cos 2x + c_2\,e^{2x}\sin 2x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (f) $y(x) = c_1\,e^{\frac{3}{2}x}\cos\frac{x}{2} + c_2\,e^{\frac{3}{2}x}\sin\frac{x}{2}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (g) $y(x) = c_1\,e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + c_2\,e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$

2. (a) $y(x) = 2(1+2x)e^{-2x}\,.$
   (b) $y(x) = \frac{1}{2}\,e^{x}(2\cos 2\sqrt{2}x - \sqrt{2}\sin 2\sqrt{2}x)\,.$
   (c) $y(x) = \frac{1}{3}(e^{1-x} - e^{4-4x})\,.$
   (d) $y(x) = -e^{\frac{5}{2}x}\left(\cos\frac{\sqrt{3}}{2}x - \sqrt{3}\sin\frac{\sqrt{3}}{2}x\right).$
   (e) $y(x) = e^{3x}\cos 5x - \frac{4}{5}\,e^{3x}\sin 5x\,.$

3. (a) $y(x) = c_1 + c_2 e^{x} - 8x - 4x^2 - x^3\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (b) $y(x) = c_1 + c_2 e^{3x} + \frac{1}{4}(1-2x)e^{x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (c) $y(x) = c_1 + c_2 e^{-x} - (1 + e^{-x})\ln(1 + e^{x})\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (d) $y(x) = c_1 + c_2 e^{x} - \frac{1}{2}x\sin x - \frac{1}{2}(1+x)\cos x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$

4. (a) $y(x) = c_1 e^{-2x} + c_2 e^{4x} + 1 - 3x + 2x^2\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (b) $y(x) = c_1 e^{-5x} + c_2 + x + x^2\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (c) $y(x) = c_1\cos 3x + c_2\sin 3x + x^2\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (d) $y(x) = c_1 e^{-4x} + c_2 e^{-3x} + e^{x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (e) $y(x) = c_1 e^{-4x} + c_2 e^{-3x} - (1-x)e^{-3x},\ \ c_1, c_2 \in \mathbb{R}\,.$
   (f) $y(x) = c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x + 2e^{x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (g) $y(x) = c_1 e^{-2x} + c_2 e^{3x} - \frac{1}{2}e^{2x}(5\cos x - 3\sin x)\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (h) $y(x) = c_1 e^{x}\cos 2x + c_2 e^{x}\sin 2x - xe^{x}\cos 2x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (i) $y(x) = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x - \frac{x}{2\sqrt{3}}\sin\sqrt{3}x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (j) $y(x) = c_1 e^{\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + c_2 e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x + e^{x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$

5. (a) $y(x) = (c_1 + c_2 x)e^{3x} - \frac{1}{2}\,e^{3x}\ln(1+x^2) + xe^{3x}\,\mathrm{artg}\,x\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (b) $y(x) = c_1 e^{x} + c_2 e^{-x} + e^{-x}\sin e^{x} - \cos e^{x}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (c) $y(x) = c_1 e^{x} + c_2 e^{3x} - \frac{1}{2}\,e^{2x} + \frac{1}{2}\,e^{3x}\,\ln(1+e^{-x}) - \frac{1}{2}\,e^{x}\,\ln(1+e^{x})\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (d) $y(x) = c_1 e^{2x} + c_2 e^{3x} + \frac{1}{6} + e^{2x} - e^{3x}\,\mathrm{artg}(e^{-x}) - \frac{1}{2}\,e^{2x}\,\ln(1+e^{-x})\,,\ \ c_1, c_2 \in \mathbb{R}\,.$
   (e) $y(x) = c_1 e^{3x} + c_2 e^{6x} + \frac{1}{9}\,e^{6x}\,e^{e^{-3x}}\,,\ \ c_1, c_2 \in \mathbb{R}\,.$

6. (a) $y(x) = 4e^{x} + (x-2)e^{2x}\,.$
   (b) $y(x) = -2e^{-4x} + 3e^{-2x} - xe^{-2x} + x^2\,e^{-2x}\,.$
   (c) $y(x) = \frac{1}{32}\,(e^{-5x} + 2xe^{-5x} - e^{-x} + 2x\,e^{-x})\,.$
   (d) $y(x) = \cos 2x + \sin 2x + 2x^2 - 1\,.$
   (e) $y(x) = \frac{1}{2}\,\sin x + \frac{\sqrt{3}}{6}\,\sin\sqrt{3}x\,.$