Formule di trigonometria

Funzioni circolari

$$\sin x = \sum_{n \geq 0}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n \geq 0}(-1)^n \frac{x^{2n}}{(2n)!}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\text{ctg}\, x = \frac{\cos x}{\sin x}$$

Relazione fondamentale

$$\sin^2 x + \cos^2 x = 1$$

Formule di addizione e sottrazione

$$\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y$$

$$\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$

$$\text{ctg}\,(x \pm y) = \frac{\text{ctg}\, x \cdot \text{ctg}\, y \mp 1}{\text{ctg}\, y \pm \text{ctg}\, x}$$

Formule di duplicazione

$$\sin 2x = 2\ \sin x \cdot \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\ \cos^2 x - 1 = 1 - 2\ \sin^2 x$$

$$\tan 2x = \frac{2\ \tan x}{1 - \tan^2 x}$$

$$\text{ctg}\, 2x = \frac{\text{ctg}^2 x - 1}{2\ \text{ctg}\, x}$$

Formule di bisezione

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\operatorname{ctg} \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

Formule di prostaferesi

$$\sin x + \sin y = 2 \, \sin \frac{x + y}{2} \cdot \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \, \cos \frac{x + y}{2} \cdot \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \, \cos \frac{x + y}{2} \cdot \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \, \sin \frac{x + y}{2} \cdot \sin \frac{x - y}{2}$$

$$\tan x + \tan y = \frac{\sin(x + y)}{\cos x \cdot \cos y}$$

$$\tan x - \tan y = \frac{\sin(x - y)}{\cos x \cdot \cos y}$$

$$\operatorname{ctg} x + \operatorname{ctg} y = \frac{\sin(x + y)}{\sin x \cdot \sin y}$$

$$\operatorname{ctg} x - \operatorname{ctg} y = -\frac{\sin(x - y)}{\sin x \cdot \sin y}$$

Formule di Werner

$$\sin x \cdot \sin y = \frac{1}{2} \left[\cos(x - y) - \cos(x + y) \right]$$

$$\sin x \cdot \cos y = \frac{1}{2} \left[\sin(x + y) + \sin(x - y) \right]$$

$$\cos x \cdot \sin y = \frac{1}{2} \left[\cos(x + y) + \cos(x - y) \right]$$

Espressioni razionali del seno e del coseno in funzione di $t = \tan \frac{x}{2}$ (per $x \neq (2n+1)\pi, n \in \mathbb{Z}$)

$$\sin x = \frac{2t}{1+t^2}, \qquad \cos x = \frac{1+t^2}{1+t^2}$$